Optimización de Funciones Irrestrictas

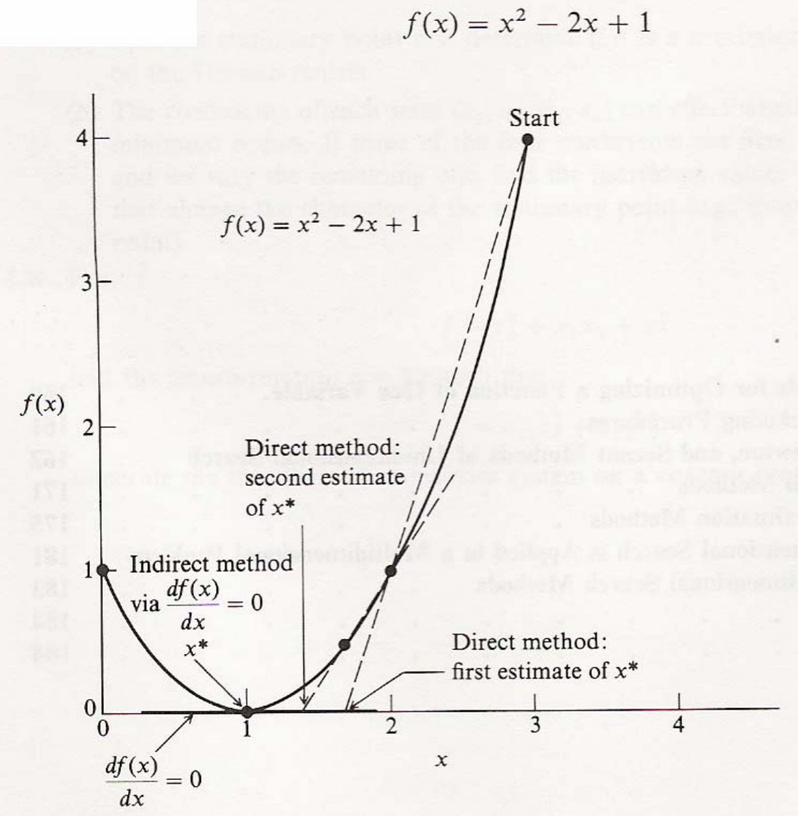


Figure 5.1 Direct versus indirect methods of finding a minimum.

<u>Métodos directos:</u> sólo se usan valores de la función objetivo, se detiene cuando $f(x^{k+1})-f(x^k)<\varepsilon$

<u>Métodos Indirectos</u>

$$\frac{\partial f(\mathbf{x})}{\partial x_1} = f_{x_1}(\mathbf{x}) = 0$$

$$\frac{\partial f(\mathbf{x})}{\partial x_2} = f_{x_2}(\mathbf{x}) = 0$$

$$\vdots$$

$$\frac{\partial f(\mathbf{x})}{\partial x_n} = f_{x_n}(\mathbf{x}) = 0$$

Método de Newton

$$x^{k+1} = x^k - \frac{f'(x^k)}{f''(x^k)}$$

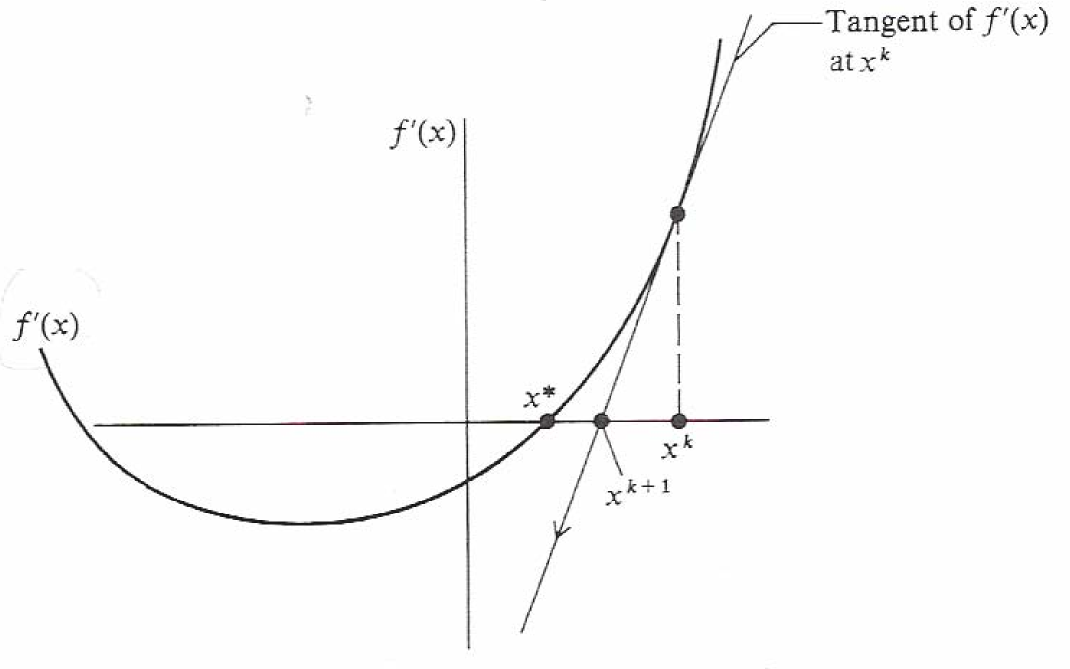


Figure 5.2 Newton's method applied to the solution of $f'(x) = 0$.

Método Quasi Newton

$$x^{k+1} = x^k - \frac{[f(x+h) - f(x-h)]/2h}{[f(x+h) - 2f(x) + f(x-h)]/h^2}$$

Método de la Secante

$$f'(x^k) + m(x - x^k) = 0$$

$$m = \frac{f'(x^q) - f'(x^p)}{x^q - x^p}$$

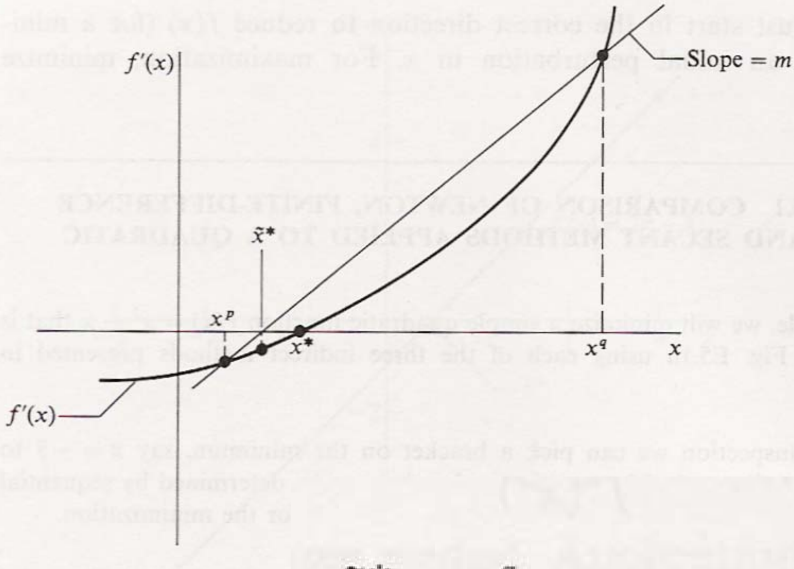
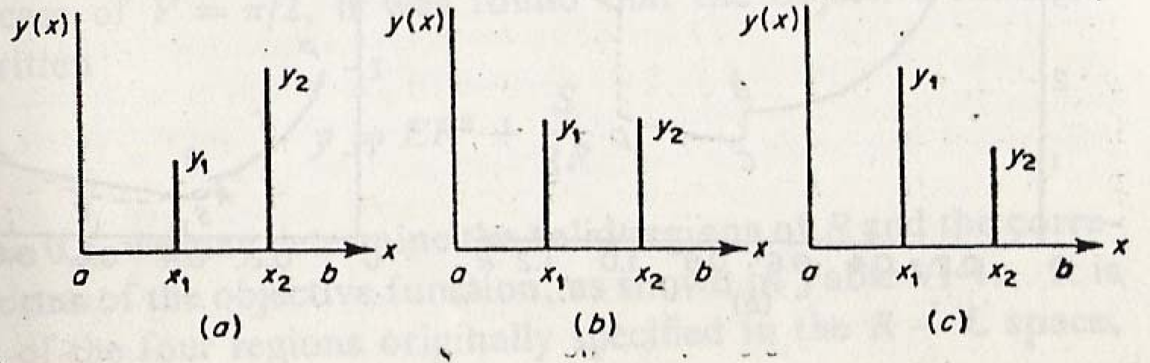


Figure 5.3 Secant method

$$\tilde{x}^* = x^q - \frac{f'(x^q)}{[f'(x^q) - f'(x^p)]/(x^q - x^p)}$$

Métodos directos ó de eliminación de regiones

Fig. VI-3 Unimodality in numerical search. (a) $y(x_1) < y(x_2)$; (b) $y(x_1) = y(x_2)$; (c) $y(x_1) > y(x_2)$.



Funciones Restringidas de una única variable se deben resolver sobre el rango permitido de las variables de la forma

$$x_L \leq x^L$$

Método de Búsqueda Preplaneada

$$x_i = a + \frac{i(b-a)}{N+1}$$

where $i = 1, 2, \ldots, N$.

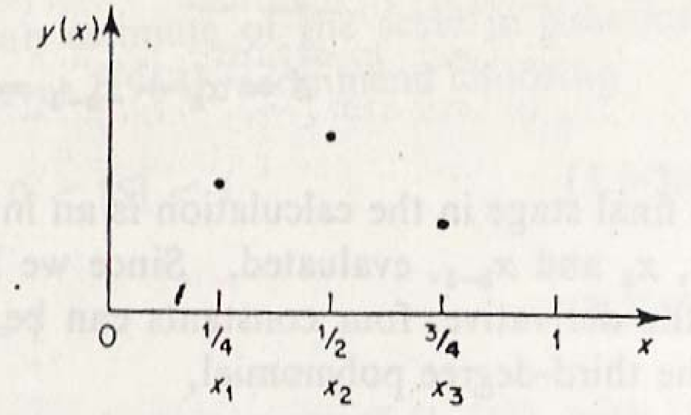


Fig. VI-9 Three-experiment preplanned search.

Medida de la Efectividad de Búsqueda

$$\alpha = L_N / L_0$$

Sabiendo que L0(1+N) es el largo de cada intervalo en el que se dividió (a,b) inicial

$$\alpha = \frac{L_N}{L_0} = \frac{2}{N+1}$$

$$N \geq \frac{2}{\alpha} - 1 \qquad \text{for integer } N$$

Métodos de Búsqueda Secuencial Uniforme

Se estudia como los resultados de un pequeño conjunto de experimentos influyen en el resultado del próximo conjunto

Búsqueda Secuencial con dos experimentos igualmente espaciados por ciclo

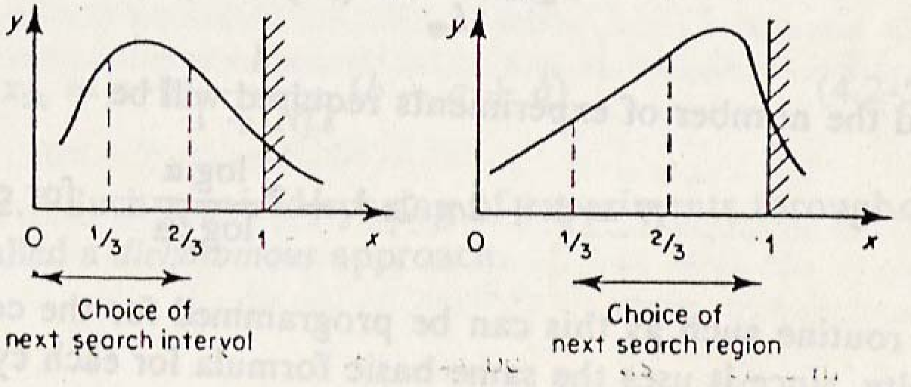
$$x_1 = a + \tfrac{1}{3}L_0 \quad \text{and} \quad x_2 = a + \tfrac{2}{3}L_0$$

$$L_N = L_{2m} = (\tfrac{2}{3})^m L_0$$

$$\alpha = \frac{L_N}{L_0} = (\tfrac{2}{3})^m = (\tfrac{2}{3})^{N/2}$$

$$N = 2m \geq 2\,\frac{\log \alpha}{\log \tfrac{2}{3}}$$

Fig. VI-10 Search using two experiments per cycle.



Búsqueda Secuencial con Tres experimentos igualmente espaciados por ciclo

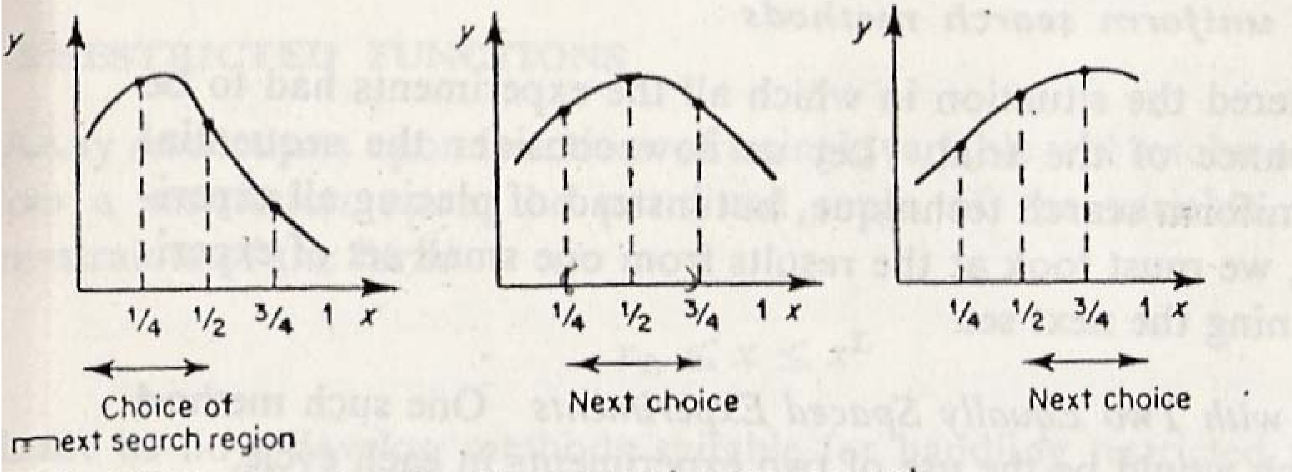


Fig. VI-11 Search using three experiments per cycle.

$$\alpha = \frac{L_N}{L_0} = (1/2)^m = (1/2)^{(N-1)/2}$$

$$N = 1 + 2m \geq 1 + 2\,\frac{\log \alpha}{\log 1/2}$$

Búsqueda Secuencial intervalos de medidas irregulares

Búsqueda basada en la Secuencia de los números de Fibonacci

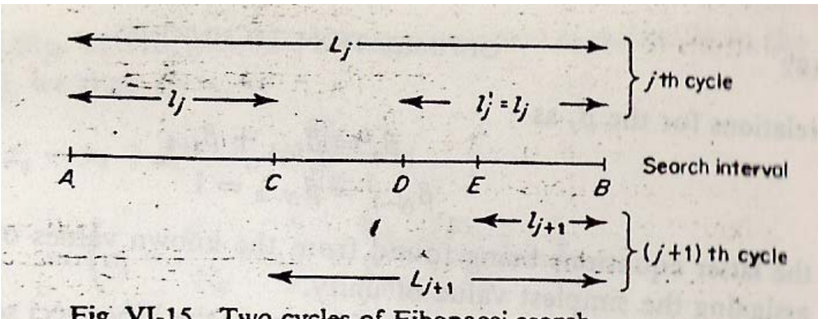


Fig. VI-15 Two cycles of Fibonacci search.

TABLE VI-8 SOME NUMBERS IN THE FIBONACCI SERIES

n	F_n
0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89
11	144
12	233
13	377
14	610
15	987
16	1,597
17	2,584
18	4,181
19	6,765
20	10,946

Step 1 Placing the first two experiments

$$l_1 = \frac{F_{N-2}}{F_N} L_1$$

$$x_1 = a_1 + l_1$$

$$x_2 = b_1 - l_1 = a_1 + L_1 - \left(\frac{F_{N-2}}{F_N}\right) L_1 = a_1 + \frac{F_{N-1}}{F_N} L_1$$

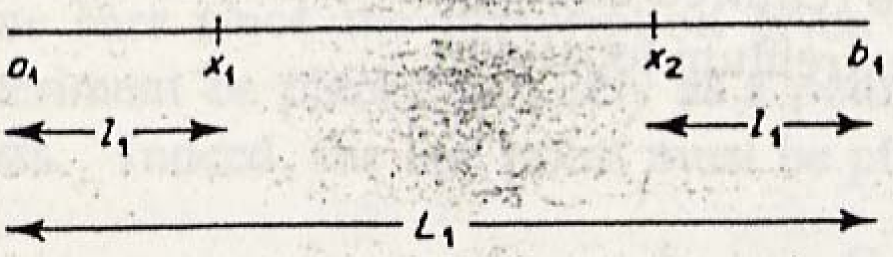


Fig. VI-16 The placing of the first two experiments.

Step 2 The new subinterval of search

$$L_2 = L_1 - l_1 = b_1 - x_1$$

$$l_k = \frac{F_{N-(k+1)}}{F_{N-(k-1)}} L_k$$

$$L_2 = L_1 - L_1 \frac{F_{N-2}}{F_N} = L_1 \left(1 - \frac{F_{N-2}}{F_N}\right) = L_1 \frac{F_{N-1}}{F_N}$$

Exactitud de Búsqueda: $\alpha = \frac{1}{F_N}$

El experimento final en la búsqueda: F0 / F2=1 /2

Métodos de aproximación polinomial

Interpolación cuadrática

$$f(x) = a + bx + cx^2$$

$$\tilde{x}^* = -\frac{b}{2c}$$

$$f(x_1) = a + bx_1 + cx_1^2$$
$$f(x_2) = a + bx_2 + cx_2^2$$
$$f(x_3) = a + bx_3 + cx_3^2$$

$$\tilde{x}^* = \frac{1}{2}\left[\frac{(x_2^2 - x_3^2)f_1 + (x_3^2 - x_1^2)f_2 + (x_1^2 - x_2^2)f_3}{(x_2 - x_3)f_1 + (x_3 - x_1)f_2 + (x_1 - x_2)f_3}\right]$$

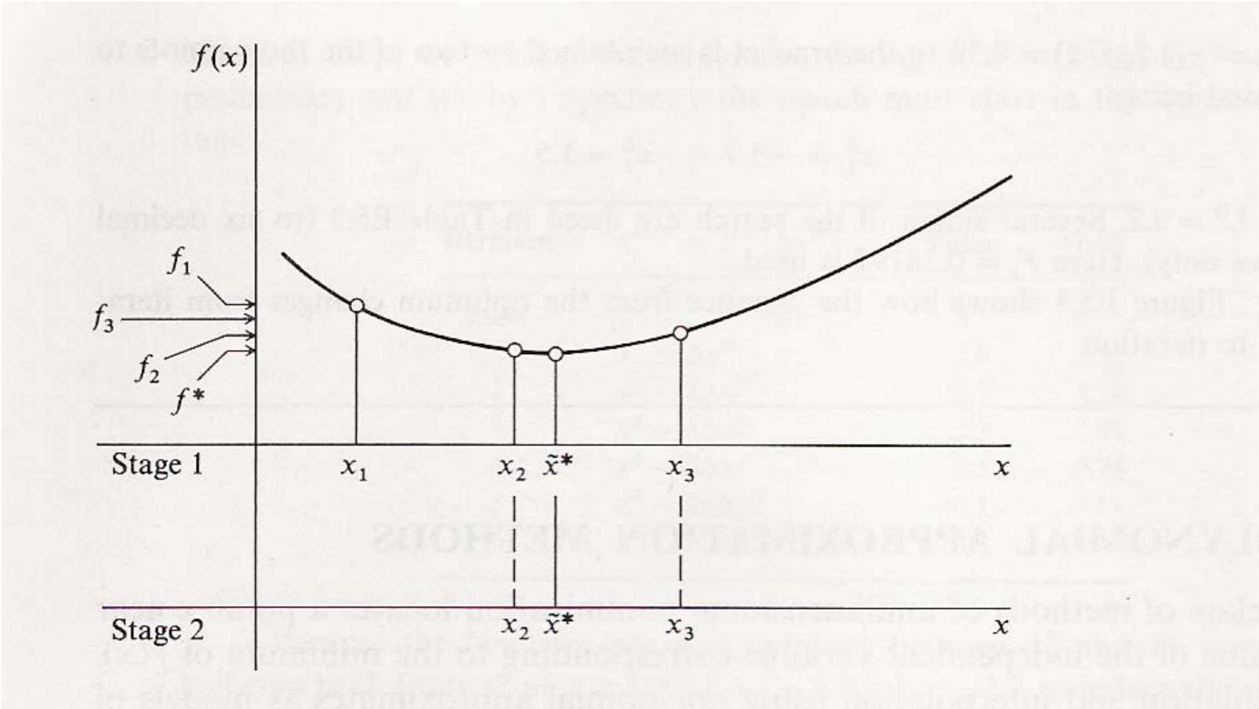


Figure 5.7 Two stages of quadratic interpolation.

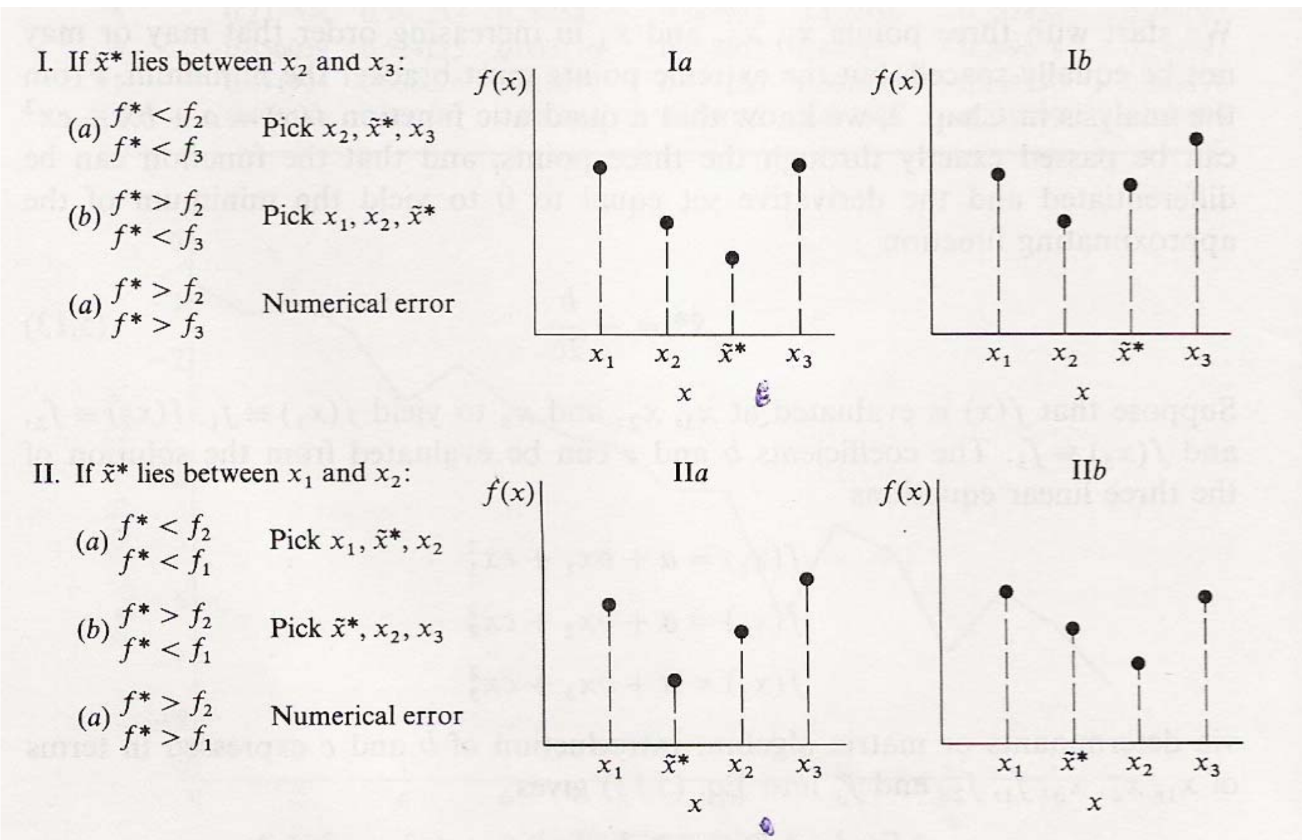


Figure 5.8 How to maintain a bracket on the minimum in quadratic interpolation.

Interpolación cúbica

$$f(x) = a_1 x^3 + a_2 x^2 + a_3 x + a_4$$

$$\mathbf{X} = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ x_2^3 & x_2^2 & x_2 & 1 \\ x_3^3 & x_3^2 & x_3 & 1 \\ x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix}$$

$$\mathbf{F}^T = [f(x_1) \quad f(x_2) \quad f(x_3) \quad f(x_4)]$$

$$\mathbf{A}^T = [a_1 \quad a_2 \quad a_3 \quad a_4]$$

$$\mathbf{F} = \mathbf{XA}$$

$$\mathbf{A} = \mathbf{X}^{-1}\mathbf{F}$$

$$\tilde{x}^* = \frac{-2a_2 \pm \sqrt{4a_2^2 - 12a_1a_3}}{6a_1}$$

$$\tilde{x}^* = x_2 - \left[\frac{f'_2 + w - z}{f'_2 - f'_1 + 2w}\right](x_2 - x_1)$$

where $z = 3[f_1 - f_2]/[x_2 - x_1] + f'_1 + f'_2$
$w = [z^2 - f'_1 \cdot f'_2]^{1/2}$

Para minimización $x_1 < x_2,\ f'_1 < 0, \text{ and } f'_2 > 0$

Bibliografía:

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* Beveridge and Schechter, "Optimization: Theory and Practice“, Mac Graw Hill